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EFFICIENT TRADING MECHANISMS WITH PRE-PLAY COMMUNICATION

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SOCIAL SCIENCE WORKING PAPER 693

January 1989

Abstract

This paper studies the problem of designing efficient trading mechanisms when players may engage in pre-play communication. It is well known that equilibrium behavior can be affected, sometimes drastically, if players have the opportunity to exchange messages prior to playing some particular game. We investigate the relationship between efficiency, pre-play communication, and unique implementation. We identify a class of simple mechanisms which are immune to pre-play communication and show that any *incentive efficient* allocation can be uniquely implemented by such a mechanism.

1. Introduction

This paper studies the problem of designing efficient trading mechanisms when players may engage in pre-play communication. It is by now well known that equilibrium behavior can be affected, sometimes drastically, if players have the opportunity to exchange messages prior to playing some particular game (Crawford and Sobel [1982]). Applications of this basic insight include bilateral bargaining (Farrell and Gibbons [1988], Matthews and Postlewaite [1988]), adoption of new technology (Farrell and Saloner [1987]), presidential veto threats (Matthews [1987]) polls and straw votes (Ordeshook and Palfrey [1988]), and voluntary public good provision (Palfrey and Rosenthal [1988]). A central finding has been that communication can enlarge the set of equilibria relative to the game with no communication, but cannot reduce the set of equilibria. The above papers identify economically interesting aspects of the comparison between these two sets equilibria in specific settings.

From the standpoint of mechanism design theory, this effect of communication raises important issues. On one hand, the expansion of equilibria is very encouraging, since outcomes which were generated in equilibrium without communication can still be achieved, and, in addition, it may be possible to achieve even more desirable outcomes than those possible without communication. This suggests that the implementation of optimal allocations may be possible using relatively "simple" trading games which are preceded by pre-play communication. On the other hand, the results suggest that communication may lead to the possibility of undesirable equilibrium outcomes which were not possible without communication. Thus, while communication may introduce better equilibrium outcomes, it may worsen the (full) implementation problem.

In our view, it is often reasonable to suppose that traders can engage in non-binding communication. Consequently, if we aim to analyze the outcomes that can be attained by some institution, it may be very important for the set of outcomes of the institution to be robust to such communication. In this paper, we propose one way of resolving this problem in a wide class of models. We show that there is a close relationship between efficiency, pre-play communication, and unique (or full) implementation. Our main result is that it is possible to design simple trading mechanisms which yield interim

efficient utility allocations as unique equilibrium outcomes with and without pre-play communication.

One of the best known models to which our results apply is that of non-cooperative bargaining. The bargaining problem with incomplete information has been extensively studied in the literature, and two approaches can be distinguished: the *mechanism design* approach, and the *institutional* approach. The mechanism design approach seeks to characterize all possible bargaining outcomes which can arise from trading mechanisms. The institutional approach, on the other hand, characterizes the outcomes to particular trading schemes. The k-double auction has played a central role in this development as have various sequential trading mechanisms.

A major contribution of the mechanism design literature has been the characterization of *efficient* bargaining outcomes. Myerson and Satterthwaite [1983] and Williams [1987] characterize *ex-ante* efficient outcomes to the bargaining problem. Wilson [1985] and Satterthwaite and Williams [1987] provide a characterization of *interim* efficient outcomes.

In light of these results, a question of considerable interest is whether some intuitive and "reasonable" trading mechanisms yield efficient outcomes. Myerson and Satterthwaite [1983] showed that the equilibrium analyzed by Chatterjee and Samuelson [1983] in the split the difference double auction with uniform distributions was *ex-ante* efficient. Satterthwaite and Williams [1987] demonstrate that for an open set of parameters, the k-double auction has an *ex-ante* efficient equilibrium. However, it is also known that double auctions generally have a continuum of inefficient equilibria (Leininger, Linhart and Radner [1986], Satterthwaite and Williams [1987]).

In a recent paper, Matthews and Postlewaite [1987] examine the consequences of allowing unmediated pre-play communication, or "cheap talk," in k-double auctions. They show that adding this feature to the model has dramatic consequences for the set of equilibria. First, by allowing traders to correlate their strategies in ways impossible without communication, they show that communication dramatically expands the set of equilibria so that, among others, efficient allocations are equilibrium outcomes. Second, they show that the set of equilibrium outcomes with cheap-talk is independent of the parameter k . We are able to show that this bargaining game with

communication can be altered slightly to produce efficient and (essentially) unique outcomes.

The general topic of this paper, then, is efficient mechanism design with pre-play communication. While much of our analysis is motivated by the double auction, our results extend to any setting with private values, independent types, and a divisible good. In this class of models, we first show that any interim efficient utility allocation can be made the *unique* outcome to a mechanism *even if pre-play communication is allowed*. In other words, such payoffs can be fully implemented in a cheap-talk proof manner. The mechanism we construct in the proof is a very simple augmentation of a direct mechanism. Second, we examine the set of equilibria which can be attained by allowing communication in arbitrary given trading mechanisms (such as the double auction). Here, too, we obtain a positive result on the set of utility allocations which can be attained by the mechanism with pre-play communication. We show that any utility allocation which is interim efficient relative to this set can be fully implemented in a cheap talk proof manner by a very simple augmentation of the underlying mechanism. We illustrate this result in the context of the double auction.

Because we are identifying allocations that can be uniquely implemented, our results are clearly related to those in the literature on full Bayesian implementation (see Postlewaite and Schmeidler [1986], Palfrey and Srivastava [1985], Mookherjee and Reichelstein [1987], Jackson [1987]). This literature has identified very general conditions under which an incentive compatible allocation can be made the unique Bayesian equilibrium outcome to a mechanism. Our results show that in the class of models examined in this paper, interim efficient allocations always satisfy these conditions in the sense that the interim utility of any such allocation can be made the unique equilibrium outcome to a mechanism. The two essential differences between the full implementation approach and that followed here are that first, we require uniqueness in utility rather than in outcomes, and second, we require immunity to pre-play communication.

In the next Section, we discuss the problems of multiple equilibria and pre-play communication in the specific context of the k-double auction. The model analyzed in the paper is presented in Section 3. In Section 4, we

analyze the possibility of implementing interim efficient allocations without communication, while our main results on mechanism design with pre-play communication are contained in Section 5. Extensions of our results and a discussion of the relationship between the results presented here and the literature on full implementation are contained in Section 6.

2. Example : Double Auctions

Consider the following bargaining problem between a seller and a buyer. The seller has one unit of a good which he may sell to the buyer. The valuation of the seller is drawn independently from a uniform distribution on $[0,1]$, and that of the buyer is also drawn independently from a uniform distribution on $[0,1]$. Let v_s denote the valuation of the seller, v_b that of the buyer, and let y be the payment made by the buyer to the seller. The problem is to decide when to trade, and the payment the seller should receive.

One institution which could be used by the parties is the *k-double auction*. The rules of the game are as follows : both players simultaneously submit bids, say b and s for the buyer and seller respectively. If $b \geq s$, then the good is awarded to the buyer and the seller receives a monetary transfer $y = ks + (1-k)b$. No trade takes place and no money changes hands if b is less than s . Let $b(v_b)$ be the strategy of the buyer, $s(v_s)$ that of the seller.

This simple double auction has a plethora of equilibria. Chatterjee and Samuelson [1983] show that the following strategies form a Bayesian equilibrium when $k = 1/2$:

$$\begin{aligned} b(v_b) &= (1/12) + (2/3)v_b \\ s(v_s) &= (1/4) + (2/3)v_s. \end{aligned}$$

Thus, trade takes place if $v_b \geq (1/4) + v_s$, and the payment made by the buyer is $(1/6) + (1/3)(v_b + v_s)$. As shown by Myerson and Satterthwaite [1983], this allocation is ex-ante efficient (in the sense of Holmstrom and Myerson [1981]) since it maximizes the expected gains from trade among all incentive compatible, individually rational mechanisms.

Leininger, Linhart, and Radner [1986] show that there is another family of equilibria in which the traders employ step functions as strategies. Included in this family are *single-price equilibria*, defined as follows. For a fixed $z \in [0,1]$, define

$$b(v_b) = \begin{cases} z & \text{if } v_b \geq z \\ 1 & \text{otherwise} \end{cases} \quad s(v_s) = \begin{cases} z & \text{if } v_s \leq z \\ 1 & \text{otherwise} \end{cases}$$

For any $z \in [0,1]$, these strategies form a Bayesian equilibrium. More

generally, there also exist multiple price equilibria in which bidding strategies are step functions. These equilibria are generally not efficient although there is experimental evidence that they may well arise in the actual play of the double auction (see Radner and Schotter [1986]).

Satterthwaite and Williams [1987] show that if $k \in (0,1)$, then there is also a continuum of *differentiable* equilibrium strategies, which are also typically inefficient (in various senses). These diverse equilibria produce different allocations and different interim utilities.

Finally, Matthews and Postlewaite [1988] have shown that there are yet many more equilibria if we allow for the possibility of non-binding pre-play communication. In fact, these communication equilibria are independent of the parameter k in the k -double auction. As an example of their construction, consider the 0-double auction, which is the "buyers-bid" auction. Without communication, this auction has a unique equilibrium in undominated strategies. The equilibrium outcome is interim efficient and, from the point of view of the seller, is the *best* possible incentive compatible, individually rational allocation (Riley and Zeckhauser [1983]). The following strategies constitute a (sequential) equilibrium to the game with communication: the buyer sends the message z , where z maximizes $(v_b - z)F(z)$, where the prior distribution over v_s is F . In the actual play of the double auction, both players follow the single price equilibrium strategy with respect to z . The outcome here is best for the buyer even though the game being studied is supposed to be the best for the seller.

This paper attempts to address the following issues, raised by the variety of problems described above:

- (1) Can an interim efficient outcome be made the *unique* equilibrium to a game?
- (2) When can such outcomes be made cheap-talk proof so that uniqueness is not undermined by pre-play communication ?
- (3) If a particular game has an more than one outcome, can the game be augmented slightly to select a particular equilibrium outcome ?
- (4) When can this be done in a cheap-talk proof manner ?

Questions 1 and 3 are analyzed in Section 4, while questions 2 and 4 are studied in Section 5.

3. The Model

There are I agents, indexed by i , and a set A of alternatives. We assume that A is a metric space. T^i denotes the set of possible types for agent i , and $F^i(t_{-i})$ denotes the distribution function on T^{-i} . Note that we are assuming *independent* types.

Given a type t_i , an alternative $a \in A$, and a real number y^i , the utility of agent i is given by $U^i(a, y^i, t_i)$, where y^i is a monetary transfer to agent i , and can be positive or negative. We assume that utility is strictly increasing in the transfer, that U^i is measurable with respect to the Borel sigma-algebra on $A \times R$, and that for any compact subset of $A \times R$, the range of U^i lies in a compact subset of the real line, R . The specification of the utility function contains the assumption of *private values* in that the utility function of i does not depend on t_{-i} . We discuss more general (common value) specifications in Section 6. Let $P(A \times R^I)$ denote the set of all probability measures on the Borel sets of $A \times R^I$.

Definition 1: An allocation rule is a function $p : T \rightarrow P(A \times R^I)$ such that there exists a compact subset of $A \times R^I$ containing the support of $p(t)$ for all t .

By assuming that U^i measurable and bounded on compact sets and that the supports of all measures in the range of p lies in the same compact set, we ensure that we do not encounter any integrability problems when defining expected utility. We interpret $p(a, y, t)$ as the joint probability of alternative a and the transfers $y = (y^1, \dots, y^I)$. The reader may wish to restrict p so that the transfers always sum to zero; however, assuming that allocations are "balanced" in this way is not needed for any of our results. We will denote by $p^i(t)$ the marginal distribution on A and the i 'th copy of R .

Many problems of interest are covered by this model, and many of the models used to study them employ the assumption of *transferable utility*, which is the special case in which $U^i(a, y, t_i) = U^i(a, t_i) + y^i$. These include the following.

Example 1: Auctions

Suppose a seller wishes to auction an object, and there are I buyers. Let t_i denote the value of the object to buyer i . Let a_0 denote the alternative "the seller keeps the object", and a_j the alternative "buyer j gets the object". Then, $p(a_j, t)$ is the probability that j gets the object, and $y^j(t)$ is the payment made by buyer j . If all the agents are risk neutral, this corresponds to a special case of the model in Myerson [1981]. Maskin and Riley [1986] and others have examined optimal auctions when buyers are risk averse.

Example 2: Bilateral Bargaining

This is the problem discussed in Section 2. Let a_1 denote the alternative "the seller keeps the object" and let a_2 be the alternative "the buyer receives the object". Let T^1 be the set of possible valuations for the seller and T^2 those for the buyer, and let $y^1(t) = -y^2(t)$ be the payment received by the seller. This is the model of Chatterjee and Samuelson [1981], Myerson and Satterthwaite [1983], and also studied by Leininger, Linhart, and Radner [1986] and Satterthwaite and Williams [1987].

Example 3: Public Goods

Let a_1 denote the alternative "the public good is built" and a_2 the alternative "the public good is not built". Let t_i denote the value of the public good to agent i , and interpret the monetary transfers to be taxes paid by the agents. The public good costs c to produce. Efficiency requires production if and only if the sum of the individual valuations exceeds c . The design problem is to construct a rule for making balanced side payments so that efficient production is achieved in equilibrium. This is the public goods allocation problem studied by D'Aspremont and Gerard-Varet [1979], Laffont and Maskin [1982], Mailath and Postlewaite [1988], and others.

Example 4: Oligopoly

There are I firms producing a homogeneous good with (different) constant average costs of production. The cost to firm i of producing q_i units is $t_i q_i$. Given a vector of outputs $Q = (q_1, \dots, q_I)$, industry revenue is $R(Q)$. Efficiency requires only the lowest cost firm to produce all the output. The

alternatives are vectors of output, $a = (q_1, \dots, q_I)$, utility is given by $U^i(a, t_i) = -t_i q_i$, and the transfers, y^i , sum to $R(Q)$ in a balanced mechanism. This problem was originally studied by Roberts [1983, 1985], and has been explored further by Cramton and Palfrey [1987] and Kihlstrom and Vives [1987].

Example 5: Dissolving a Partnership

Several individuals own shares of an asset they value differently. Each partner's (constant) marginal valuation of the asset is t_i . The asset valuations are private information, and the design problem is to find a set of balanced transfers which enable efficient dissolution. Efficiency requires the partner with the highest valuation to own all the shares. Here, an alternative $a = (\alpha_1, \dots, \alpha_I)$ with $\alpha_i \geq 0$ and $\sum \alpha_i = 1$ specifies the distribution of shares, and $U^i(a, t_i) = t_i \alpha_i$. This model is studied by Cramton, Gibbons, and Klemperer [1987].

As is well known, the set of allocation rules which can be made equilibrium outcomes to games are restricted by incentive compatibility conditions. We develop these next. For any $\tau_i \in T^i$, let

$$V^i(p, t_i, \tau_i) = \int_{A \times R} \int_{T^{-i}} U^i(a, y^i, t_i) p^i(da, dy^i, \tau_i, t_{-i}) dF^i(t_{-i})$$

Definition 2: An allocation rule p is *incentive compatible* if for all i and $t_i \in T^i$,

$$V^i(p, t_i, t_i) \geq V^i(p, t_i, \tau_i) \text{ for all } \tau_i \in T^i.$$

We will write $V^i(p, t_i, t_i) = V^i(p, t_i)$.

In the analysis that follows, we do not address problems of individual rationality. However, our results also apply if allocations are constrained by individual rationality constraints as well as by the usual incentive constraints.

Definition 3: A *reduced form allocation rule*, P , is a collection of functions $P = (P^1, \dots, P^I)$, with $P^i: T^i \rightarrow P(A \times R)$.

A reduced form allocation rule, P , can be constructed from an allocation rule, p , by defining

$$P^i(B \times C, t_i) = \int_{T^{-i}} p^i(B \times C, t_i, t_{-i}) dF^i(t_{-i}) \text{ for all measurable sets } B \subseteq A, C \subseteq R.$$

Then, it is easily verified that

$$V^i(p, t_i) = \int_{A \times R} dU^i(a, y^i, t_i) P^i(da, dy^i, t_i).$$

The assumptions of private values and independence have the following consequence which is central to our analysis. Suppose that agent i is faced with the choice of reporting an element of T^i . If the agent reports τ_i , outcome a is chosen and the agent receives a transfer y^i with probability $P^i(a, y^i, \tau_i)$. Then, incentive compatibility implies that when faced with these outcomes, agent i will truthfully report t_i . This follows from the fact that

$$V^i(p, t_i, \tau_i) = \int_{A \times R} dU^i(a, y^i, t_i) P^i(da, dy^i, \tau_i).$$

Mechanisms and equilibrium

A *mechanism* is $\mu = (M, g)$, where $M = M^1 \times M^2 \times \dots \times M^I$, and g is a function $g : M \rightarrow P(A \times R^I)$, again satisfying the condition that there is a compact subset of $A \times R^I$ containing the support of $g(m)$ for all $m \in M$. The set M^i is the *message space* of agent i , and g is called the *outcome function*. For each $m \in M$, $g(m)$ yields a probability measure on $A \times R^I$, specifying the joint distribution of alternatives and transfers. We denote by $g^i(m)$ the marginal distribution on A and the i 'th component of R^I .

Definition 4: A *strategy* for i is a function $\sigma^i: T^i \rightarrow M^i$.

Given a strategy profile $\sigma = (\sigma^1, \dots, \sigma^I)$, the interim utility to i when of type t_i is given by

$$W^i(\sigma, t_i) = \int_{A \times R} \int_{T^{-i}} U^i(a, y^i, t_i) g^i(da, dy^i, \sigma(t)) dF^i(t_{-i})$$

Definition 5: σ is a *Bayesian equilibrium* if for i and t_i ,

$$W^i(\sigma, t_i) \geq W^i(\sigma^{-i}, \alpha^i, t_i) \text{ for all } \alpha^i: T^i \rightarrow M^i$$

We are now in a position to define what we mean by implementation in utility.

Definition 6: An allocation rule p is *essentially implementable* if there exists a mechanism (M, g) , with at least one Bayesian equilibrium, such that for every Bayesian equilibrium, σ , for all i and t_i ,

$$V^i(p, t_i) = W^i(\sigma, t_i)$$

4. Efficient and Unique Implementation without Communication

The main focus of this paper is on interim efficient allocation rules (see Holmstrom and Myerson [1983]). One of the central unanswered questions in mechanism design theory is the relationship between efficient allocations and fully implementable allocations. The motivation behind much of recent work on fully implementation is that many many incentive compatible allocations may not be uniquely achievable by any mechanism. This can happen if they fail to satisfy a condition called Bayesian monotonicity (Postlewaite and Schmeidler [1986], Palfrey and Srivastava [1989]). In this Section, we show that this is not a problem in the class of environments described in Section 3. Indeed, we show here that there is a close connection between interim efficiency and full implementability. Interim efficient allocations are defined as follows.

Definition 1: p is *interim efficient* if p is incentive compatible and there does not exist another incentive compatible allocation rule, say q , such that $V^i(q, t_i) \geq V^i(p, t_i)$ for all i and t_i and with strict inequality for at least one i and t_i .

The next result shows that any interim efficient allocation rule is essentially implementable. The proof relies heavily on the reduced form being incentive compatible.

Theorem 1: Let p be interim efficient. Then, it is essentially implementable.

Proof: A proof is given in the Appendix. Here, we provide a particularly simple proof for the case of transferable utility, i.e. when the utility function takes the form $U^i(a, t_i) + y^i$. Consider the reduced form allocations corresponding to p ,

$$P^i(t_i) = \int_R \int_T p^i(dy^i, t_i, t_{-i}) dF^i(t_{-i}), \text{ the marginal distribution on } A,$$

and

$$Y^i(t_i) = \int_R \int_A \int_{T^{-i}} y^i p^i(da, dy^i, t_i, t_{-i}) dF^i(t_{-i}), \text{ the expected transfer.}$$

Then, it is easily verified that

$$V^i(p, t_i) = \int_A U^i(a, t_i) P^i(da, t_i) + Y^i(t_i), \text{ and that}$$

$$V^i(p, t_i, \tau_i) = \int_A U^i(a, t_i) P^i(da, \tau_i) + Y^i(\tau_i).$$

Incentive compatibility implies that "truth" is an equilibrium in the direct mechanism, where $M^i = T^i$ and $g = p$. Consider the indirect game in which the message space of i is $T^i \times [0,1]$. The outcomes are modified as follows :

(1) If, for each i , $(t_i, 0)$ is reported, the outcome is as under the direct mechanism, i.e. $p(t)$.

(2) If all agents other than i report a zero in the second component of their message space while i reports (τ_i, ϵ) with $\epsilon > 0$, then the outcome is chosen according to the distribution $P^i(\tau_i)$, i receives the transfer $Y^i(\tau_i)$, and every $j \neq i$ receives the transfer $-[Y^i(\tau_i) - \epsilon]/(I-1)$. Thus, if i alone deviates from reporting a zero, he receives the reduced form gamble of his reported type but receives a lower (by ϵ) transfer. The transfer to i is divided equally among the other agents.

(3) If more than one agent reports a strictly positive number, then the agent reporting the lowest positive number plays the role of i above. If there is a tie, then the agent with the lowest index and the lowest number plays the role of i above.

Now, observe that honestly reporting your type and reporting zero is a Bayesian equilibrium, by the fact that the reduced form gambles are incentive compatible. Furthermore, note that all equilibria involve reporting zero, since it always pays to reduce your positive number. Finally, if there is another equilibrium, it must yield everyone at least as great an interim

payoff as p at every type, since otherwise some type of some agent would be worse off and should ask to play his original gamble and pay slightly more than in the original gamble, i.e. report (t_i, ϵ) . Since the original allocation is interim efficient, all equilibrium utility allocations yield this efficient interim payoff to all agents.

The proof for the more general class of models considered in this paper contains exactly the same intuition as the case of transferable utility. Since this is notationally more cumbersome, we provide the proof in the Appendix. ■■■

The augmentation we have performed essentially converts the direct mechanism into a "unanimity" game : any agent has the right to veto a proposed equilibrium outcome and revert to his reduced form gamble by paying ϵ . The power of this result can be illustrated by considering in detail the bilateral trading problem discussed in Section 2. The Chatterjee-Samuelson equilibrium in the split the difference double auction with uniform distributions produces the following (deterministic) allocation rule:

$$q(v_b, v_s) = \begin{cases} 1 & \text{if } v_b \geq (1/4) + v_s \\ 0 & \text{otherwise} \end{cases}$$

$$y(v_b, v_s) = \begin{cases} (1/6) + (1/2)(v_b + v_s) & \text{if } v_b \geq (1/4) + v_s \\ 0 & \text{otherwise} \end{cases}$$

One can interpret this allocation rule, $p = (q, y)$, simply as a direct mechanism in which truth telling is an equilibrium. The interesting property of this allocation rule is that it is ex-ante efficient, and thus interim efficient. However, there are many inefficient equilibria to the direct mechanism defined by p . One of these is the following, which yields the same outcomes as the single price equilibrium in the k -double auction with $z = 1/2$:

$$\beta(v_b) = \begin{cases} 5/8 & \text{if } v_b \geq 1/2 \\ 0 & \text{otherwise} \end{cases} \quad \sigma(v_s) = \begin{cases} 3/8 & \text{if } v_s \leq 1/2 \\ 1 & \text{otherwise} \end{cases}$$

One can easily see that this is not an equilibrium to the mechanism constructed in the proof of Theorem 1, since the buyer would deviate by reporting a small $\epsilon > 0$ for a wide range of types. For instance, if $v_b = 3/8$, the buyer receives interim utility of 0 in the single price equilibrium and $\int_0^{1/8} [3/8 - (1/2)[3/8 + v_s]] dv_s > 0$ in the Chatterjee-Samuelson equilibrium. It is easily verified that any $\epsilon < 13/768$ improves the payoff to the buyer of type $v_b = 3/8$ and thus breaks the single price equilibrium.

Efficiency Relative to a Mechanism

Theorem 1 shows that in the class of models described in Section 3, a simple augmentation of a direct mechanism essentially implements any interim efficient allocation rule. In this Section, we ask whether or not a given trading mechanism can be augmented so as to eliminate "undesirable" equilibria. By undesirable we mean any equilibrium which is Pareto dominated by another equilibrium. It is well known that many games have multiple, Pareto-ranked equilibria. What we are suggesting here is that very simple institutional features may allow us to focus on the preferred equilibria. To motivate this analysis, consider the k-double auction. As discussed in Section 2, Satterthwaite and Williams [1987] and Leininger, Linhart and Radner [1986] have shown that these auctions have large multiplicities of equilibria, most of them inefficient. These results raise the following question: if we view the k-double auction as a reasonable trading mechanism, is there a simple modification of the auction which retains much of the simplicity of the original mechanism but eliminates all outcomes which are inefficient *relative to those attainable by a k-double auction* ? We now answer this question affirmatively.

Let μ be a mechanism, and let E_μ denote the set of equilibria to μ . Then, $\sigma \in E_\mu$ is an *interim efficient equilibrium* of μ if there does not exist $\sigma' \in E_\mu$ such that $W^i(\sigma', t_i) \geq W^i(\sigma, t_i)$ for all i and t_i with strict inequality for at least one i and t_i .

Definition 8: $\mu' = (M', g')$ is an *augmentation* of $\mu = (M, g)$ if there exist sets N^i , $i=1, 2, \dots, I$ such that
(i) $M'^i = M^i \times N^i$, so $M' = M \times N$
(ii) there exists $n \in N$ such that $g'(m, n) = g(m)$ for all $m \in M$.

This definition is written differently but is equivalent to that given by Mookherjee and Reichelstein [1987].

Theorem 2: If σ is interim efficient equilibrium of μ , then there exists an augmentation of μ , say μ' , such that every equilibrium of μ' yields interim

utility equal to that from σ .

Proof: A proof is given in the Appendix. Here, we again consider the very simple case of transferable utility.

Let σ be an interim efficient equilibrium of $\mu = (M, g)$. We construct μ' from μ along the lines of the construction in Theorem 1. Let the message space of each i be $M^i \times [0, 1]$. The outcomes are specified as follows.

(1) If no agent reports a positive number, then the outcome is the same as that given by g .

(2) If only one agent, say i , reports (m^i, ϵ) with $\epsilon > 0$, then the outcome is chosen according to the distribution $P^i(m^i)$, and agent i receives a transfer $Y^i(m^i) - \epsilon^i$, where

$$P^i(m^i) = \int_{\mathbb{R}^A} \int_{T^{-i}} g^i(dy^i, m^i, \sigma^{-i}(t_{-i})) dF^i(t_{-i})$$

and

$$Y^i(m^i) = \int_{\mathbb{R}^A} \int_{T^{-i}} y^i g^i(da, dy^i, m^i, \sigma^{-i}(t_{-i})) dF^i(t_{-i}).$$

Every $j \neq i$ receives the transfer $-[Y^i(m^i) - \epsilon]/(I-1)$.

(3) If more than one agent reports a positive number, then the agent with the lowest number plays the role of i above; again, ties are broken by choosing the agent with the lowest index.

Since σ is an equilibrium to (M, g) , the argument employed in Theorem 1 implies that $(\sigma, 0)$ is an equilibrium to the augmented mechanism. It is similarly shown that all equilibria to the augmented mechanism must yield interim utility equal to that from σ . ■■■

It is easy to see how the augmentation eliminates single price equilibria in the k -double auction. Consider, for example, such an equilibrium

corresponding to $z = 1/2$. Then, if values are uniformly distributed between 0 and 1, the expected payoff to the buyer in this equilibrium is $(1/2)(v_b - 1/2)$ if $v_b \geq 1/2$ and zero otherwise, while the expected payoff in the efficient equilibrium identified by Chatterjee and Samuelson [1983] is $(1/2)(v_b - 1/4)^2$ if $v_b \geq 1/4$ and zero otherwise. Thus, the buyer should deviate from the single price strategy if $v_b \in (1/4, 3/4)$.

6. Implementation with pre-play communication

This section investigates implementation when players may communicate with each other prior to playing the game. It is by now well-known that the possibility of unmediated communication, or *cheap talk* (Farrell [1988]), generally produces abundant equilibria, since it allows players to use communication to transmit information and/or correlate their strategies in ways that were not available when the original form of the mechanism did not include these early "payoff irrelevant" stages. This explosion of equilibria presents interesting and important problems from the standpoint of mechanism design (Farrell [1983]). On the positive side, it suggests that "good" equilibria may emerge from "intuitive" mechanisms which are augmented by early rounds of cheap talk among players. This possibility is indeed encouraging, as one of the common criticism of the usefulness of mechanism design theory has been that the implementing mechanisms produced by current theoretical tools are often complicated and impractical. Consideration of very simple mechanisms combined with unmediated communication therefore seems like a promising direction to pursue in search of "realistic" mechanism design. On the other hand, if the resulting good equilibrium is only one of many, then the usual "full implementation" criticisms apply. (See Postlewaite and Matthews [1987] for a more detailed statement of the issues raised by this difficulty). This multiplicity problem is further compounded by the fact that often none of the standard refinement techniques succeed in eliminating unwanted equilibria. (See Farrell [1988] and Matthews, Okuno-Fujiwara, and Postlewaite [1988] for extensive discussions of the relationship between communication and equilibrium refinements).

This section addresses this multiplicity problem and proposes a potential solution. Suppose that there is a direct mechanism, μ , to be played, and suppose that there is an equilibrium σ that produces an interim efficient allocation p . Further suppose that the players cannot be prevented from communicating prior to playing μ . The question we pose is : under what conditions can μ be augmented to μ' so that p is a unique equilibrium outcome of μ' and no (finite) amount of cheap-talk can undermine p , in the sense of introducing extraneous, unwanted equilibrium outcomes to μ' ?

The answer we provide is very encouraging, and is closely related to the analysis in the previous section. There we constructed a class of augmentations to uniquely implement the interim utility of p whenever p is interim efficient. In this section we show that this augmentation is cheap-talk proof. In order to state this result precisely, we need to introduce some formal definitions.

Let $\mu = (M, g)$ be any mechanism. We wish to consider a class of augmentations of μ that involve a type of communication related to (but not as general as) Forges' (1988) "communication devices" and Myerson's (1986) "communication mechanisms." These mechanisms are $K+2$ stage games. In the first stage (stage 0), nature moves and privately assigns all players their types according to some known distribution. No player has a move at this time and all players privately observe their own type. At stage k , $1 \leq k \leq K$, all players simultaneously and publicly broadcast messages, say $m_k^i \in M_k^i$, and then receive the messages sent by all other players. For simplicity the set of allowed messages for each player, M_k^i , is assumed not to depend on the past messages sent by all the players. Finally, in stage $K+1$, μ is played out and an allocation is determined according to g . Note that the outcome function depends only on the messages contained in μ , and not on any messages sent during the communication rounds.

We refer to the K intermediate stages as a *K-stage Public Communication Procedure*, $C_k = \{M_1^1, \dots, M_1^I, \dots, M_K^1, \dots, M_K^I\}$. Given any mechanism $\mu = (M, g)$, any K -stage communication procedure C_k , and any joint distribution over types, F , we call the corresponding $K+2$ stage game, μ_k , a *K-stage public communication extension of μ* . This is similar to what Farrell [1983] calls a communication version of μ . We discuss more general communication possibilities in Section 6.

Finally, let X_μ^* equal the set of Bayesian equilibrium outcomes to μ and let X_μ^{**} equal the set of all allocations that are Bayesian equilibrium outcomes to some K -stage communication game for μ for some K . We make a point of examining Bayesian equilibria instead of sequential equilibria of the communication game. The reason is simple. We are interested in proving that our augmented games are immune to cheap talk. Since all sequential equilibria are also Bayesian equilibria, if we can show that (essentially) no Bayesian

equilibria are added by pre-play communication then we have also shown that no sequential equilibria are (essentially) added.

Since we are investigating the Bayesian equilibria of finite stage public communication games, we will restrict attention to the normal form. A strategy for agent i in μ_K is a sequence of functions $(\sigma_1^i, \dots, \sigma_{K+1}^i)$ with

$$\begin{aligned}\sigma_1^i &: T^i \rightarrow M_1^i \\ \sigma_2^i &: T^i \times M_1 \rightarrow M_2^i \\ \sigma_K^i &: T^i \times M_1 \times \dots \times M_{K-1} \rightarrow M_K^i \\ \sigma_{K+1}^i &: T^i \times M_1 \times \dots \times M_K \rightarrow M^i\end{aligned}$$

A Bayesian equilibrium is then defined exactly as in Definition 3.

We call X_μ^{**} the set of *Public Communication Equilibria* of μ . Observe that $X_\mu^* \subseteq X_\mu^{**}$. We say that μ is *cheap-talk proof* if $X_\mu^* = X_\mu^{**}$. Let V_μ^* and V_μ^{**} be the sets of interim utility allocations associated with X_μ^* and X_μ^{**} respectively. We say that μ is *essentially cheap-talk proof* if $V_\mu^* = V_\mu^{**}$. We are now ready to state the main result of this section.

Theorem 3: Let p be interim efficient. Then it is essentially implementable using an essentially cheap-talk proof mechanism.

Proof: We already know it is essentially implementable using the mechanism μ' in Theorem 1. To see that it is essentially cheap-talk proof, suppose not. Then, there is a K -stage communication game and an equilibrium, say σ , producing an allocation q . Since p is interim efficient, $V^i(q, t_i) \leq V^i(p, t_i)$ for all i and t_i and there exists $\epsilon > 0$ such that for some i and some $t_i \in T^i$ $V^i(q, t_i) < V^i(p, t_i) - \epsilon$. Consider the following alternative strategy, s , for player i :

$$\begin{aligned}s_k &= \sigma_k^i \text{ for all } k \leq K \\ s_{K+1}^i(\tau_i, m_K) &= \sigma_{K+1}^i(\tau_i, m_K) \text{ for all } m_K \text{ and all } \tau_i \neq t_i \\ s_{K+1}^i(t_i, m_K) &= (t_i, \epsilon/2) \text{ for all } m_K.\end{aligned}$$

This alternative strategy improves i 's interim payoff by at least $\epsilon/2$ when i is of type t_i and does not affect his payoffs at any other type. Hence, σ is not an equilibrium. ■■■

Given a mechanism μ , we denote by E_{μ}^* the subset of X_{μ}^* which is not interim dominated by any other element of X_{μ}^* . Similarly, E_{μ}^{**} is the set of allocations in X_{μ}^{**} which are not interim dominated by any other element of X_{μ}^{**} . We call E_{μ}^{**} the *public communication frontier* of μ . It is the set of allocations which are achievable as equilibria of some K-stage communication extension of μ and which are not interim Pareto dominated by any other public communication equilibrium of μ . One can prove, as we do below, that for any $p \in E_{\mu}^{**}$, there exists an augmentation of μ , say μ' , so that p is essentially implemented by μ' and μ' is essentially cheap-talk proof.

This provides a strengthening of Theorem 2 in much the same way as Theorem 3 strengthened Theorem 1. However, while construction in Theorem 3 was the same as in Theorem 1, the construction in Theorem 4 is different from the construction in Theorem 2. The reason for this is that we are implementing different allocations in Theorems 2 and 4. Theorem 2 applies to implementation of allocations that are efficient relative to the set of equilibrium allocations of a mechanism. Theorem 4 applies to implementations of allocations that are efficient relative to the entire set of all public communication equilibria of a mechanism. The latter set may include some allocations that cannot arise as equilibria of the mechanism alone; they only arise if some pre-play communication is added. It is not the case that an exact analog of Theorem 2 holds for cheap-talk proof implementation. Typically, there may be allocations in E_{μ}^{**} which interim Pareto dominate E_{μ}^* . When this is the case, those inferior allocations of μ cannot be essentially implemented using an essentially cheap-talk proof mechanism. Furthermore, for those allocations x such that $x \in E_{\mu}^{**}$ but $x \notin E_{\mu}^*$, simply augmenting μ as before will not work. Instead, it is necessary to consider an augmentation of a K-stage communication extension of μ , as in the proof of the next theorem.

Theorem 4: Let $p \in E_{\mu}^{**}$ for some $\mu = (M, g)$. Then p is essentially implementable using an essentially cheap-talk proof mechanism.

Proof: Since $p \in E_{\mu}^{**}$, there exists a K-stage public communication extension of

μ , say $\mu_K = (M_1, \dots, M_K, M, g)$, and an equilibrium to μ_K , say σ , such that $g(\sigma) = p$.

Consider the following augmented mechanism, derived from μ_K :

$$\begin{aligned}\mu'_K &= (M_1 \times [0,1], M_2, \dots, M_K, g'), \text{ where} \\ g'((m_1, 0), m_2, \dots, m_K, m) &= g(m) \\ g'((m_1, \epsilon), m_2, \dots, m_K, m) &= p_\epsilon^i(m)\end{aligned}$$

where $\epsilon = (\epsilon^1, \dots, \epsilon^I)$, and i is the agent with the lowest index among the set of agents reporting the smallest positive number, and p_ϵ^i is constructed in exactly the same way as in the proof of Theorem 2.

It is straightforward to verify that the following is an equilibrium strategy: $s_1^i(t_i) = (\sigma_1^i(t_i), 0)$, $s_k^i(t_i, m_{k-1}) = \sigma_k^i(t_i, m_{k-1})$ for $k = 2, \dots, K+1$, all i and t_i . and that $g(s) = p$, implying that $p \in X_{\mu'_K}^*$.

We now demonstrate that the interim utilities from p are the only equilibrium interim payoffs in μ'_K , and that μ'_K is cheap talk proof.

To see the former, note that no equilibrium to μ'_K can ever involve a message at stage one in which some agent reports a positive number. Consequently, following exactly the same argument as in the proofs of Theorems 1-3, no equilibrium can ever yield a payoff which is interim dominated by that from p for any i at any t_i , since this agent should report a small positive number and receive approximately the payoff from p . Suppose, then, that there is an equilibrium to μ'_K , say s , which interim dominates the payoff from p . Then, we claim that $s \in X_{\mu'_K}^*$. To see this, we simply observe that when playing s , no agent is ever using a positive number. Thus, when we eliminate the strategies which involve using a positive number, i.e. change the game from μ'_K to μ_K , s continues to be an equilibrium. This contradicts the fact that $p \in E_{\mu}^{**}$.

To prove the latter, consider any L stage public communication extension of

μ' , say $\mu'_{LK} = (N_1, \dots, N_L, M_1 \times [0, 1], M_2, \dots, M_K, M, g')$. Again, p is an equilibrium outcome to μ'_{LK} . Noting that no equilibrium to μ'_{LK} can involve any agent reporting a positive number, the same arguments as above imply that no equilibrium payoff to μ'_{LK} can be interim worse than p . Finally, since no equilibrium in μ'_{LK} involves reporting a positive number, any equilibrium to μ'_{LK} is also an equilibrium to μ_{LK} (defined in the obvious way). Therefore, $p \in E_{\mu}^{**}$ implies that no equilibrium to μ'_{LK} can interim dominate p . ■■■

6. Extensions

In Section 5, we only considered public communication. However, our results apply to much more general communication structures. In particular, the results can be extended in (at least) three ways. First, we could allow M_k^i to depend on previous messages. Second, we could allow for private communication between players. Third, we could incorporate mediated communication, which would allow for messages to be "garbled" or "filtered." These extensions would require more cumbersome notation without changing the nature of the arguments in Theorems 3 and 4.

The assumptions of private values and independence are central to the construction employed in the proof of our main results, since they imply that the reduced form is incentive compatible. This need not hold with either common values (in which case U^i depends on t_{-i} as well as on t_i) or in the dependent case (where $F^i(t_{-i})$ depends on t_i). To see why, consider, for example, the dependent case with transferable utility, so that the distribution on T^{-i} given t_i is written $F^i(t_{-i}|t_i)$. Let

$$P^i(t_i, \tau_i) = \int_{\mathbb{R}} \int_{T^{-i}} p^i(dy^i, \tau_i, t_{-i}) dF^i(t_{-i}|t_i)$$

and

$$Y^i(t_i, \tau_i) = \int_{\mathbb{R}} \int_A \int_{T^{-i}} y^i p^i(da, dy^i, \tau_i, t_{-i}) dF^i(t_{-i}|t_i).$$

Now, it is easy to see that in order to maintain incentive compatibility, agent i *cannot* generally be offered reduced form probability and transfer functions which depend only on his *reported* type. Incentive compatibility now only says that if, at true type t_i , i is faced with $P^i(t_i, \tau_i)$ and $Y^i(t_i, \tau_i)$ if he reports τ_i , then he will report $\tau_i = t_i$. It does not say that i will report truthfully if faced with $P^i(\tau_i, \tau_i)$ and $Y^i(\tau_i, \tau_i)$. What is important, then, is that the reduced form probability functions and transfers depend only on the reported type of the agent and this reduced form be incentive compatible.

A similar problem, that of defining the appropriate reduced form, appears with common values, where U^i depends not only on t_i but also on t_{-i} . However, there is a large class of cases in which this is possible. This class includes the "revision effects" considered by Myerson [1981], and requires the utility function to be separable in the following sense. Suppose

$$U^i(a, t) = \phi^i(a, t_i) + \psi^i(a, t_{-i}).$$

Then, with independent types, essentially the same argument as in Theorem 1 applies. To see this in the case of transferable utility, let

$$\text{Let } P^i(t_i) = \int_{\mathbb{R}} \int_{T^{-i}} p^i(dy^i, t_i, t_{-i}) dF^i(t_{-i})$$

and

$$\begin{aligned} Y^i(t_i) = & \int_{\mathbb{R}} \int_A \int_{T^{-i}} [p^i(da, dy^i, t_i, t_{-i}) - P^i(da, t_i)] \psi^i(a, t_{-i}) dF^i(t_{-i}) \\ & + \int_{\mathbb{R}} \int_A \int_{T^{-i}} y^i p^i(da, dy^i, t_i, t_{-i}) dF^i(t_{-i}). \end{aligned}$$

Then,

$$V^i(p, t_i) = \int_A \int_{T^{-i}} U^i(a, t) P^i(da, t_i) dF^i(t_{-i}) + Y^i(t_i).$$

Here, we have converted the "common value" part of utility into its monetary equivalent. It is easy to show that incentive compatibility implies that the reduced form is also incentive compatible. Thus, we can use the same augmentation as in the proof of Theorem 1 to essentially implement any interim efficient allocation rule.

Finally, we discuss the relationship between our results and the literature on full Bayesian implementation. Let F denote a set of allocation

rules. Then, F is *fully implementable* if there is a mechanism (M, g) such that the set of equilibrium outcomes to (M, g) equals F . It is known that in order for F to be fully implementable, it must satisfy a condition called Bayesian Monotonicity (see Postlewaite and Schmeidler [1986], Palfrey and Srivastava [1989]). With independent types, this is defined as follows.

Definition 9: F satisfies *Bayesian Monotonicity* if for every collection of functions $\alpha = \{\alpha^1, \dots, \alpha^I\}$ with $\alpha^i : T^i \rightarrow T^i$, $i=1, 2, \dots, I$, and for every $p \in F$, if $p_\alpha \notin F$ then there exists an allocation rule q such that $V^i(q_\alpha, t_i) > V^i(p_\alpha, t_i)$ for some i and t_i and $V^i(p, t'_i) \geq V^i(q, t'_i)$ for all i and t'_i , where $p_\alpha(t) = p(\alpha(t))$ and $q_\alpha(t) = q(\alpha(t))$ for all t .

Theorem 1 immediately implies that if p is interim efficient, then the set of allocation rules defined by $F = \{p_\alpha : V^i(p_\alpha, t_i) = V^i(p, t_i)\}$ is fully implementable, and thus satisfies Bayesian Monotonicity. This can also be verified directly as follows:

(1) Consider any α such that $V^i(p_\alpha, t_i) < V^i(p, t_i) - \epsilon$ for some i, t_i and some $\epsilon > 0$. Define the constant allocation rule $q(t_{-i}, t'_i) = P_\epsilon^i(t_i)$ for all t_{-i} and t'_i , where $P_\epsilon^i(t_i)$ is constructed in the proof of Theorem 1. Then, q satisfies the conditions of Bayesian Monotonicity.

(2) Consider any α such that $V^i(p_\alpha, t_i) \geq V^i(p, t_i)$ for all i and t_i with strict inequality somewhere. Then, p_α cannot be incentive compatible since p is interim efficient. Therefore, there must exist i, t_i and τ_i such that $V^i(p_\alpha, t_i, \tau_i) > V^i(p_\alpha, t_i, t_i)$. Define $q(t_{-i}, t'_i) = p(t_{-i}, \alpha^i(\tau_i))$ so that q does not depend on player i 's type. Then, $V^i(q_\alpha, t_i) = V^i(p_\alpha, t_i, \tau_i) > V^i(p_\alpha, t_i)$, while incentive compatibility of p implies that $V^i(p, t'_i) \geq V^i(q, t'_i)$ for all t'_i , so that q satisfies the conditions of Bayesian Monotonicity. This argument is along the lines of Theorem 4.4 of Mookherjee and Reichelstein [1987].

APPENDIX

Proof of Theorem 1:

Given a reduced form allocation rule P^i and a number $\epsilon > 0$, define P_ϵ^i by : for all $t_i \in T^i$, for all (measurable) sets $B \subset A$, $C \subset R$, $P_\epsilon^i(B \times C, t_i) = P^i(B \times C_\epsilon, t_i)$ where $C_\epsilon = \{z \in R : z + \epsilon \in C\}$. Thus, $P_\epsilon^i(t_i)$ is the reduced form allocation rule corresponding to P^i in which the monetary transfers to agent i have been reduced by ϵ everywhere. Define the function $h_i : A \times R \rightarrow A \times R^I$ by

$$h_i(a, y^i) = (a, -y^i/(I-1), -y^i/(I-1), \dots, y^i, -y^i/(I-1), \dots, -y^i/(I-1))$$

where y^i is the i 'th component in the range of h_i .

Finally, given P_ϵ^i , define the measure $p_\epsilon^i(t) \in P(A \times R^I)$ by

$$p_\epsilon^i(B \times C^1 \times \dots \times C^I, t) = P_\epsilon^i(h_i^{-1}(B \times C^1 \times \dots \times C^I), t_i)$$

for all measurable sets $B \subseteq A$, $C^i \subseteq R$.

Note that incentive compatibility implies that truth is an equilibrium in the direct game. Consider the game in which the message space of i is $T^i \times [0, 1]$. The outcomes are as follows :

If, for each i , $(t_i, 0)$ is reported, the outcome is as under the direct mechanism, i.e. $p(t)$.

If all agents other than i report a zero in the second component of their message space while i reports (τ_i, ϵ) with $\epsilon > 0$, then the outcome is $p_\epsilon^i(\tau_i, t_{-i})$. (Note that $p_\epsilon^i(\tau_i, t_{-i})$ does not depend on t_{-i}). Thus, if i alone deviates from reporting a zero, he gets his reduced form gamble, but pays ϵ more. The transfer to i is divided equally among the other agents by the construction of the function h_i .

If more than one agent reports a strictly positive number, then the agent reporting the lowest number plays the role of i above. If there is a tie,

then the agent with the lowest index and the lowest number plays this role.

Now, observe that honestly reporting your type and reporting zero is a Bayesian equilibrium. This follows from incentive compatibility because for any $\epsilon \in (0,1]$, $V^i(p, t_i, t_i) \geq V^i(p, t_i, \tau_i) > V^i(p_\epsilon, t_i, \tau_i)$, the last inequality following from the assumption that utility is strictly increasing in the transfers.

Following the same argument as in the case of transferable utility, it is easy to see that all equilibria involve reporting zero and that every equilibrium yields the same interim utility as p . ■■■

Proof of Theorem 2:

For every measurable set $B \subseteq A$, $C \subseteq R$, define

$$P^i(B \times C, m^i) = \int_{T^{-i}} g^i(B \times C, m^i, \sigma^{-i}(t_{-i})) dF^i(t_{-i})$$

$$P_\epsilon^i(B \times C, m^i) = \int_{T^{-i}} g^i(B \times C_\epsilon, m^i, \sigma^{-i}(t_{-i})) dF^i(t_{-i}) \quad \text{and}$$

$$p_\epsilon^i(B \times C^1 \times \dots \times C^I, m) = P_\epsilon^i(h_i^{-1}(B \times C^1 \times \dots \times C^I), m^i)$$

for all measurable sets $B \subseteq A$, $C^i \subseteq R$.

We construct μ' from μ as in the proof of Theorem 1. Let the message space of each i be $M^i \times [0,1]$. The outcomes are specified as follows.

If no agent reports a positive number, then the outcome is the same as that given by g .

If all agents other than i report a zero in the second component of their message space while i reports (m^i, ϵ) with $\epsilon > 0$, then the outcome is $p_\epsilon^i(m)$. (Note that $p_\epsilon^i(m)$ does not depend on m^{-i}).

If more than one agent reports a strictly positive number, then the agent

reporting the lowest number plays the role of i above. If there is a tie, then the agent with the lowest index and the lowest number plays this role.

Since σ is an equilibrium to (M, g) , the argument in Theorem 1 shows that $(\sigma, 0)$ is an equilibrium to μ' . It is similarly shown that all equilibria to the augmented mechanism must yield interim utility equal to that from σ . ■■■

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